

# NAG Toolbox for MATLAB

## f07ga

### 1 Purpose

f07ga computes the solution to a real system of linear equations

$$AX = B,$$

where  $A$  is an  $n$  by  $n$  symmetric positive-definite matrix stored in packed format and  $X$  and  $B$  are  $n$  by  $r$  matrices.

### 2 Syntax

```
[ap, b, info] = f07ga(uplo, ap, b, 'n', n, 'nrhs_p', nrhs_p)
```

### 3 Description

f07ga uses the Cholesky decomposition to factor  $A$  as  $A = U^T U$  if **uplo** = 'U' or  $A = LL^T$  if **uplo** = 'L', where  $U$  is an upper triangular matrix and  $L$  is a lower triangular matrix. The factored form of  $A$  is then used to solve the system of equations  $AX = B$ .

### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Parameters

#### 5.1 Compulsory Input Parameters

1: **uplo** – string

If **uplo** = 'U', the upper triangle of  $A$  is stored.

If **uplo** = 'L', the lower triangle of  $A$  is stored.

*Constraint:* **uplo** = 'U' or 'L'.

2: **ap**(\*) – double array

**Note:** the dimension of the array **ap** must be at least  $\max(1, n \times (n + 1)/2)$ .

The  $n$  by  $n$  symmetric matrix  $A$ , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of  $A$  must be stored with element  $A_{ij}$  in **ap**( $i + j(j - 1)/2$ ) for  $i \leq j$ ;

if **uplo** = 'L', the lower triangle of  $A$  must be stored with element  $A_{ij}$  in **ap**( $i + (2n - j)(j - 1)/2$ ) for  $i \geq j$ .

3: **b**(ldb,\*) – double array

The first dimension of the array **b** must be at least  $\max(1, n)$

The second dimension of the array must be at least  $\max(1, \text{nrhs\_p})$

The  $n$  by  $r$  right-hand side matrix  $B$ .

## 5.2 Optional Input Parameters

1: **n** – **int32 scalar**

$n$ , the number of linear equations, i.e., the order of the matrix  $A$ .

*Constraint:*  $n \geq 0$ .

2: **nrhs\_p** – **int32 scalar**

*Default:* The second dimension of the array **b**.

$r$ , the number of right-hand sides, i.e., the number of columns of the matrix  $B$ .

*Constraint:* **nrhs\_p**  $\geq 0$ .

## 5.3 Input Parameters Omitted from the MATLAB Interface

ldb

## 5.4 Output Parameters

1: **ap**(\*) – **double array**

**Note:** the dimension of the array **ap** must be at least  $\max(1, n \times (n + 1)/2)$ .

If **info** = 0, the factor  $U$  or  $L$  from the Cholesky factorization  $A = U^T U$  or  $A = LL^H$ , in the same storage format as  $A$ .

2: **b**(ldb,\*) – **double array**

The first dimension of the array **b** must be at least  $\max(1, n)$

The second dimension of the array must be at least  $\max(1, \text{nrhs\_p})$

If **info** = 0, the  $n$  by  $r$  solution matrix  $X$ .

3: **info** – **int32 scalar**

**info** = 0 unless the function detects an error (see Section 6).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**info** =  $-i$

If **info** =  $-i$ , parameter  $i$  had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs\_p**, 4: **ap**, 5: **b**, 6: **ldb**, 7: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

**info** > 0

If **info** =  $i$ , the leading minor of order  $i$  of  $A$  is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

## 7 Accuracy

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07gb is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04be solves  $Ax = b$  and returns a forward error bound and condition estimate. f04be calls f07ga to solve the equations.

## 8 Further Comments

The total number of floating-point operations is approximately  $\frac{1}{3}n^3 + 2n^2r$ , where  $r$  is the number of right-hand sides.

The complex analogue of this function is f07gn.

## 9 Example

```
uplo = 'U';
ap = [4.16;
      -3.12;
      5.03;
      0.560000000000000001;
      -0.83;
      0.76;
      -0.1;
      1.18;
      0.34;
      1.18];
b = [8.6999999999999999;
     -13.35;
     1.89;
     -4.14];
[apOut, bOut, info] = f07ga(uplo, ap, b)

apOut =
    2.0396
   -1.5297
    1.6401
    0.2746
   -0.2500
    0.7887
   -0.0490
    0.6737
    0.6617
    0.5347
bOut =
    1.0000
   -1.0000
    2.0000
   -3.0000
info =
```

0
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